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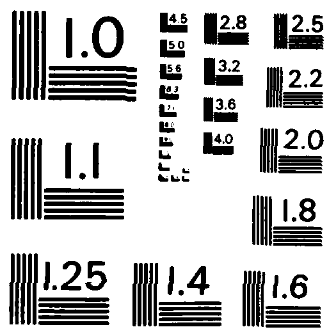
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Submitted to:

Dr. John Burns

Directorate of Mathematical and Information Sciences

Air Force Office of Scientific Research (AFSC)

Bolling Air Force Base

Washington, D.C. 20332

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ABSTRACT

This final report describes the research carried out by Professors Sanjoy K. Mitter and Bernard Levy and Mr. Yehuda Avniel and Mr. Saul Gelfand during the time period March 15, 1984 to March 15, 1985, with support extended by the Air Force Office of Scientific Research under Grant AF-AFOSR 82-0135A.

The principal investigator was Professor Sanjoy Mitter. The contract monitors were Dr. J. Burns and Dr. Marc Jacobs of the AFOSR Directorate of Mathematical and Information Sciences.

This research is concerned with fundamental aspects of filtering theory, statistical signal processing and stochastic variational problems related to estimation of Markov Random Fields. We take a viewpoint which exploits the analogies between these problems and problems of quantum and statistical physics. The proposed research is of great potential benefit to the U.S. Air Force in areas such as guidance and control, pattern recognition and image processing related to radar signals and signal processing.

TABLE OF CONTENTS

1. Introduction

This report is concerned with the following fundamental aspects of Stochastic Systems Theory:

(i) Filtering, Statistical Signal Processing and related Problems in Scattering and Inverse Scattering Theory

(ii) Theory of Markov Random Fields and related questions in Image Processing and Image Understanding

(iii) Stochastic Variational Calculus and Stochastic Adaptive Control

(iv) Parallel and Distributed Algorithms for Statistical Signal Processing

Problem areas (i) and (iii) are intimately related in the sense that stochastic control in the presence of incomplete observations has as a sub-problem non-linear filtering. Moreover, the problem of parameter identification could be considered as a special case of nonlinear filtering and the problem of optimal adaptive control (suitably formulated) can be considered as a special case of stochastic control with partial uncertain observations. One must remark that this necessitates taking a Bayesian point of view.

Stochastic Calculus of Variations could be considered as a special case of Stochastic Control. This is well known in the deterministic situation and indeed the two fields could be considered equivalent, namely, by appropriate transformation one can pass from one formulation to the other. This is, however, not so in a stochastic setting where careful distinction needs to be made between "open-loop control" (pre-programmed control) and "feedback control" (control based on past history of the observations).

This report describes work which conforms to the comprehensive proposal submitted to the Air Force Office of Scientific Research in September 1981. One of the new directions proposed was to develop a Scattering and Inverse Scattering Framework for estimation problems for random fields. Considerable progress has been made in the direction during the current period of the grant. In addition, during the current period of this grant we have initiated new research on Markov Random Field models for images for the purpose of reconstruction of surfaces from noisy data. Using a Bayesian point of view, these estimation problems lead to stochastic variational problems which can be solved using methods of simulated annealing. We have undertaken fundamental theoretical research with a view to understanding the method of simulated annealing. These methods are also particularly suited for parallel and distributed computation.

It should also be mentioned that the work proposed here is potentially of great benefit to the U.S. Air Force. Increasingly, it is being recognized that ad hoc techniques using linearization and perturbation methods are unsatisfactory and nonlinear theory is ripe for applications. The Kalman filter has played an important role in guidance and control of aerospace vehicles. However, the extended Kalman filter which is used to handle nonlinear situations is not understood from a scientific point of view and often given rise to incurable convergence difficulties. We have made some progress towards alleviating this situation. In particular, we have greatly enhanced understanding about the derivation and functioning of the Extended Kalman Filter.

The control of future aircrafts, large space structures, and aerospace vehicles is a problem of continuing importance from the point of view of designing adaptive systems that operate reliably over a wide operating envelope. Similarly, the control of advanced jet engine, whose dynamic characteristics change rapidly with operating conditions, pose difficult problems if one wishes to design a control system which accomplishes commanded thrust level changes rapidly, while maintaining fan and compressor stability margins. It would appear that an adequate theory of stochastic adaptive control will be essential to solve these problems. Due to the tremendous increase in computing power available and decrease in costs in memory size the problem of dealing with non-linearities is no longer the insurmountable obstacle it was.

Signal Processing forms an important aspect for various systems which is vital to the success of Air Force Missions. Our research leads to new algorithms and new computational structures for the solution of signal processing problems. Moreover these algorithms and structures are particularly suited to VLSI implementations and indeed often suggest novel VLSI structures.

2. Fundamental Viewpoint of our Research and Progress to Date

2.1 Introduction

Our research is concerned with the fundamental problem of statistical signal analysis, namely, the optimal probabilistic reconstruction of a signal from noisy observations of the signal. The term signal analysis encompasses such tasks as optimal processing, analyzing, and understanding of signals.

The distinctive features of this research are (i) the modelling and analysis of the signals in a form which has striking analogies to models in statistical and quantum physics; (ii) the optimal incorporation of a priori knowledge about the signal in the mathematical model; (iii) recognition of the role of symmetries (in particular, the relation between physical symmetries and the action of a symmetry group on the space of signals); (iv) algorithms for probabilistic reconstruction and analysis which permits implementation in parallel and distributed architectures.

The identification of probabilistic models of signals as models arising in physics often suggests a parallel VLSI (very large scale semi-conductor integration) implementation of the algorithm for the reconstruction of the signal. Indeed, it is believed that algorithms that are best for VLSI implementation must mirror the physics of large-scale semi-conductor integration. An example of this process might be the modelling of Stationary Gaussian Processes as generalized transmission lines.

Our recent research is also motivated by the work of Marr and Poggio [1] on vision and has as its aim the construction of a mathematical theory of early vision. According to this theory, early vision is primarily computational and is concerned with such functions as recognition of symmetries, surface reconstruction, edge-detection, and extraction of depth information. The attempt at a construction of a mathematical theory of early vision may itself shed new light upon theories of vision.

2.2 The Analogies Between Problems in Statistical (and Quantum) Physics and Optimal Probabilistic Reconstruction of Signals

In our earlier work ([2], [3], [4]) we have shown how mathematical problems of linear and non-linear filtering (where the signals are modelled as functions of Markov diffusion processes) are closely related to mathematical problems in quantum physics. The fundamental reason for this is that the representation of the estimate of the signal (from noisy observations) as a conditional expectation can be achieved using a Stochastic Feynman-Kac formula. The recursive estimation problem leads to a stochastic partial differential equation which has the interpretation as a (euclidean) field and in this viewpoint the Kalman filter plays the role of the free euclidean field (Ornstein-Uhlenbeck operator) of quantum physics. This viewpoint has led to a deepening of our understanding of non-linear filtering and new results.

We would like to argue that this viewpoint of analogies with physics is deep and leads to new insights and to solutions of new problems in the reconstruction of signals.

2.3 Images as Markov Random Fields

The use of Markov Random fields and Bayesian estimation for signal processing tasks such as image restoration and surface reconstruction has recently been proposed by Grenander, Geman (cf. for example [5]) and our own unpublished work. The simplest such model of the signal (image to be recovered) corresponds to a one-dimensional Ising ferromagnet on a finite lattice with free boundary conditions. If such a signal is observed in the presence of noise, then the corresponding mathematical model corresponds to an Ising ferromagnet with an external random magnetic field. The signal-to-noise ratio has the interpretation as the temperature of the system. Given the prior distribution of the signal and the noise, the posterior distribution can be computed using Bayes formula and is in the form of a Gibbs distribution. The maximum a posteriori probability estimate can be reduced to the minimization of an energy function and corresponds to finding the ground state of the Ising ferromagnet with external field. This problem is effectively a large integer-quadratic programming problem where the matrix has a band structure.

In recent work we have been able to decompose this problem into a sequence of one-dimensional minimization problems by using a dynamic programming recursion on the boundaries (odd bonds between neighboring cells). This decomposition corresponds to solving a sequence of estimation problems under different "scales" and is strikingly resemblant of the renormalization group approach to statistical mechanics developed by K. Wilson. In contrast to physics, however, here we are interested in obtaining the detailed structure of the random fields.

We have also used Markov random field models in conjunction with stochastic approximation (simulated annealing in the terminology of

Kirkpatrick, et al. [6]) for surface reconstruction that preserves discontinuities in images.

2.4 The S-Matrix and Estimation for Stationary Gaussian Processes

To illustrate the power of this viewpoint, we discuss some of our recent work on modelling and estimation of stationary Gaussian processes indexed by the integers. The basic idea is that the analog of the S-matrix (scattering function) representing the interaction between the past and future of a regular stationary Gaussian process is the fundamental object for performing estimation, prediction, and interpolation for Gaussian signals in additive white noise. Indeed the S-matrix can be explicitly computed and is an L^{∞}-function, unitary on the boundary of the unit-disc. If the process is strongly mixing, then this function is in the class $H^{\infty} + C$. Under some further mild assumptions, it can be shown that the S-matrix uniquely characterizes the spectral density of the process (up to a multiplicative constant). The relationship of this S-matrix to the scattering function of Lax and Phillips can be explicitly characterized.

The Hankel and Toeplitz operators induced by the S-matrix have a special role to play in this theory, and using these operators, one can solve the filtering and prediction problem and leads to a new formulation of the approximation of Gaussian processes [7]. This work uses the deep theory of approximation of Hankel operators as developed by Adamjan, Arov and Krein [8].

2.5 Estimation Theory, Statistical Signal Processing and Inverse Problems

One of the objectives of our research has been to study the links existing between linear estimation theory and inverse scattering theory, and to use this relationship to obtain efficient algorithms for solving inverse scattering problems. The algorithms that we have obtained are efficient, recursive, and operate on a layer stripping principle, whereby an unknown scattering medium is reconstructed layer by layer, in a sequential fashion. These algorithms can be viewed as the counterpart of the celebrated fast algorithms of linear estimation theory due to Levinson, Krein, Szego, and Schur, which have been used extensively by Kailath and his colleagues, Dewilde, Dym, and many others. More recently, we have also been able to show that Kalman filtering techniques can be applied to inverse scattering problems, when reflection coefficients have a rational structure. The inverse scattering methods that were obtained by this analogy with linear estimation theory were then applied to several inverse problems such as the inverse seismic or inverse resistivity problems of geophysics. The study of the relations existing between linear estimation and inverse scattering has been beneficial not only to inverse scattering, but also to linear estimation, where we have been able to solve several previously unsolved problems. For example, in [6] by using the analogy existing between the linear estimation problem for isotropic random fields, and the two-dimensional (2-D) inverse scattering problem for a potential with radial symmetry, we have obtained an efficient estimation technique for isotropic fields. This technique can be viewed as a generalization of the 1-D Levinson equations of linear prediction, but previous attempts to extend the Levinson recursions to two-dimensions had failed because they were forcing a quarter-plane or half-plane causality structure which does not exist for random fields.

Since our results are described in detail in [1]-[9], we will only outline here the main aspects of our work.

(1) In the area of inverse scattering, we have obtained in [4], [5] a new class of differential inverse scattering methods which operate on a layer stripping principle and reconstruct a scattering medium recursively, layer by layer. These inverse scattering methods generalize an algorithm* introduced by Schur in 1917 for testing the boundedness of a function which is analytic inside the unit circle. The recursions appearing in this algorithm are also identical to the so-called fast Cholesky equations for factoring a Toeplitz operator in causal times anticausal (or lower times upper triangular) form. From a more general point of view, differential inverse scattering algorithms rely on the method of propagation of singularities, and depending on whether this method is applied to two-component wave equations, to the telegrapher's equations, or to Schrodinger equations, layer stripping algorithms can be expressed in several forms. Thus in addition to the Schur algorithm that we have developed, several variations of the same technique have been proposed by Symes, Santosa and Schwetlick, Bube and Burridge, and Corones and his group, among others. Because of their recursive structure, and because the quantities that they propagate can be interpreted physically as being the waves inside the medium, layer stripping algorithms are more

convenient than traditional inverse scattering methods (introduced by Gelfand and Levitan, Marchenko, Krein, Kay and Moses, Faddeev...) which rely on integral equations. However, it was shown in [4] that differential and integral equations methods can be related from a system-theoretic point of view, by using causality. In [5] various applications of layer stripping algorithms to the reconstruction of transmission lines, inverse seismic problems, and linear estimation of stationary processes are discussed. In addition, for the case when the scattering medium that we want to reconstruct is <u>lossy</u>, and when we have access to <u>transmission data</u>, as well as scattering data, a generalization of the Schur algorithm based on two sets of coupled equations is derived and is used to reconstruct a lossy transmission line.

(2) In [7] the inverse scattering problem for the case when the reflection coefficient of the scattering medium is <u>rational</u> was considered. Several solutions of this problem have been proposed in the past, but all these solutions were quite inefficient since they relied on exploiting the rational structure of the reflection coefficient inside a Wiener Hopf equation defined for every value of the depth x at which the reflectivity function $r(x)$ of the scattering medium needs to be reconstructed. The missing concept was clearly that of a <u>state-space model</u>. In [7] by using a simple state-space representation for the left and right going waves

propagating inside the medium when the medium is probed from the left by an impulsive wave, we were able to obtain a Kalman filter-like solution for the inverse scattering problem. This solution is expressed in terms of the so-called Chandrasekhar equations of linear filtering theory.

(3) Since the fast inverse scattering algorithms for two-component wave systems described in [4], [5] are expressed in terms of the Levinson and fast Cholesky recursions, and since these recursions are usually associated with a Gram-Schmidt orthonormalization process, and perform a factorization of a Toeplitz (or Hankel) operator or of its resolvent in terms of triangular operators, it is natural to ask whether a similar orthonormalization point of view can be used to interpret the results of [4], [5]. In [9] it is shown that such an interpretation exists, but that unlike in the derivation of the Szego orthogonal polynomials, or of the continuous Krein polynomials, where a scalar spectral function was used, we need to associate a 2x2 matrix spectral function to a two-component wave system. Then by orthonormalizing the free solutions of the system with respect to this matrix, we obtain the Marchenko equations for solving the inverse scattering problem. The functions obtained by this Gram-Schmidt orthonormalization process are now 2x2 matrix orthonormal functions, and all sorts of identities, such as the Christoffel-Darboux formulas, can be obtained for these functions. The matrix operators factored by

the Levinson and fast Cholesky recursions are also put in evidence. We hope that these results will be useful in the context of linear estimation to obtain some further insight on the properties of lattice realizations of scalar stationary stochastic processes.

(4) The layer stripping inverse scattering methods developed in [4], [5] were used in [1]-[3] to solve the inverse seismic problem for one-dimensional layered media. In [1], the case of an acoustic medium was considered. In this problem, the objective is to reconstruct <u>both</u> the density $\rho(x)$ and the wave speed $c(x)$ of the medium as functions of the depth x. The Schur algorithm was used to solve this problem. By probing the medium obliquely at two different angles with plane waves, and by running two sets of recursions corresponding to these two experiments, it was shown that the density and wave speed could be reconstructed simultaneously, in a recursive way, by increasing depths. The point source case (i.e. the case when the probing wave is an impulsive spherical wave) was also considered. In this case, it was shown that by <u>slant-stacking</u>, i.e. by performing a Radon transform on the data collected by the receivers on the surface, the plane-wave data could be synthesized, and the algorithm mentioned above for two plane waves could be used. In [2], the more difficult problem when the medium is described by the <u>equations of elasticity</u> was considered. In this case two types of

waves, the pressure (P) and stress (S) waves, can propagate inside the medium and can be converted into each other at interfaces. The objective is to reconstruct the two Lame parameters $\lambda(x)$ and $\mu(x)$ of the medium and the density $\rho(x)$ as functions of depth. By performing two experiments with impulsive plane P and S waves which are obliquely incident upon the medium, in such a way that the lateral wave number is the same in both experiments, and by propagating recursively the two sets of measured waves for these two experiments for increasing depth, $\lambda(x)$, $\mu(x)$ and $\rho(x)$ can be reconstructed recursively. The only difference with [1] is that the system used to propagate the downgoing and upgoing P and S waves is a differential system of order 4 (instead of order 2 for the acoustic problem). In [3] the inverse problem for an acoustic medium probed by spherical harmonic waves, and when the reflected data is measured for all lateral wave numbers, was considered. In this case, given the data for two frequencies (instead of two angles in [1]) and by using a transformation procedure to solve the inverse resistivity problem (see (5) below), it was shown that a layer stripping technique could be used to reconstruct the density $\rho(x)$ and wave speed $c(x)$ separately.

(5) In [8] the inverse resistivity problem for a layered earth was considered. In this problem, the earth is probed by injecting some current inside the earth, and lateral potential gradients on the surface ae measured. This problem was suggested to us by Dr.

Steve Lang from Shlumberger. Since the potential is described by an elliptic equation, and not by the wave equation, this problem is not an inverse scattering problem. Nevertheless, there exists a simple mapping technique which can be used to convert the inverse resistivity problem into an equivalent inverse scattering problem, to which existing 1-D inverse scattering techniques can then be applied. In this context, it turns out that a number of previously introduced inverse resistivity techniques can be identified with classical signal processing algorithms. The mapping used to transform the inverse resistivity problem into an equivalent inverse scattering problem has also an elegant interpretation in terms of Maxwell's method of images.

(6) In [6] the problem of estimating a 2-D isotropic random field in noise given some observations over a disk of radius R was examined. The isotropy of the field and the circular symmetry of the observation geometry were exploited by expanding the field in Fourier series, and noting that the Fourier coefficient processes were uncorrelated. This implies that the original 2-D estimation estimation problem can be decomposed into a countable set of 1-D estimation problems for the Fourier coefficient processes. The filtering equation for each Fourier coefficient process turns out to be identical to the Gelfand-Levitan equation for reconstructing a circularly symmetric potential from its spectral density $r(\lambda)$, where $r(\lambda)$ is here the spectral density of the random field. The

structure of the Gelfand-Levitan equation can then be exploited to obtain some Levinson-like recursions for the optimum filter, which are very efficient, and provide therefore a simple solution of the random field estimation problem. This algorithm is the first fast algorithm ever derived in two dimensions, and a preliminary study indicates that it can also be used to solve the 2-D maximum entropy spectral estimation problem for isotropic fields.

2.6 Nonlinear Filtering and Stochastic Variational Problems

A long-standing open problem in filtering theory has been to obtain a derivation of the Extended Kalman Filter and explain its qualitative behavior. The Extended Kalman Filter is widely used in aerospace systems and is known to function very well in many situations, but is also known to exhibit divergence phenomenon in the presence of modelling errors. In the paper [1], first steps towards a rigorous derivation of Extended Kalman Filter as well as explaining its qualitative properties were taken. To obtain this result, we use the stochastic control interpretation of nonlinear filtering described in the joint paper with Wendell Fleming [2], the Morse Lemma with parameters [3], and the work of Malliavin on Stochastic Jacobi fields [4].

It was suggested that the Stochastic Control Interpretation of Nonlinear Filtering would provide the means for obtaining bounds for nonlinear filtering. A first step in this direction has been taken in the paper mentioned above. Specifically, the analog of the Fisher Information Matrix has been defined. This paper also shows how the nonlinear filtering problem is related to the identification problem. We have done further work on these issues and we are considering examples related to the phase-lock loop to understand better various lower bounds on performance that can be obtained. A joint paper with A. Moro and J. Moura is now in preparation. This work also has connections with recent work of Bobrovsky et al [5] and Adaptive Filtering.

An important open problem for the last few years has been the construction of robust filters using pathwise nonlinear filtering for observations which are unbounded. This problem has been settled in the negative by Sussmann based on earlier work by Mitter. More specifically, we have shown by example that the normalization constant becomes infinity for certain "physical" observation paths. This suggests that the normalization is important and it is necessary to construct the normalized conditioned density by examining the robust versions of the numerator and denominator of the Kallianpur-Striebel Formula after "cut-offs" have been introduced. One then has to remove the cut-off in an appropriate way. This is a familiar procedure in Statistical Mechanics but some key monotonicity properties seem to be absent in the problem under investigation.

For further progress on the qualitative behavior on nonlinear filters, it is necessary to understand the small-time and large-time behaviors of these filters. Mathematically, this problem is related to the small-time behavior of conditional diffusions and to large deviation theory. For diffusion processes there now exists a large body of theory developed by Donsker-Varadhan and Ventcel-Freidlin. What is needed is a generalization of these ideas to conditional diffusion processes, and we have begun work on that generalization.

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